ON THE GENERAL COVARIANCE IN THE BOHMIAN QUANTUM GRAVITY

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Abstract

It is shown explicitly that in the framework of Bohmian quantum gravity, the equations of motion of the space-time metric are Einstein's equations plus some quantum corrections. It is observed that these corrections are not covariant. So that in the framework of Bohmian quantum gravity the general covariance principle breaks down at the individual level. This principle is restored at the statistical level.

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I. INTRODUCTION

In the de-Broglie–Bohm interpretation of quantum mechanics [1], the quantum effects are described by the quantum potential. It has at least two peculiar properties, it is non-local, and it is able to break down the classical symmetries for an individual process while they remain valid statistically. For example, the above-mentioned problem appears for the Lorentz symmetry. So we have velocities greater than light's velocity for an individual phenomenon. [1,2]

General relativity is invariant under the general coordinate transformations. It is not clear whether this symmetry preserved after quantization or not. In the Copenhagen quantum gravity, the constraints related to this symmetry (one on Hamiltonian and three on momenta) appear weakly (in the terminology of Dirac's canonical quantization procedure). For the disscusion about the governing algebra, it is necessary to represent the operators as well-defined ones and then one obtains the commutation relations between them.

The viewpoint of the Copenhagen approach is, as usuall, statistical. For an individual description we must use Bohm's theory. In this theory, the quantum corrections to the constraints are represented by terms involving the quantum potential. But, the statistical results of this interpretation is the same as those of the Copenhagen quantum mechanics.

Essentially, in the Bohmian quantum gravity, Einstein's equations (which are the equations of motion of the space-time metric) would be modified by some expressions containing the quantum potential. In this paper we shall derive these modified Einstein's equations. As a result, the general covariance principle would be broken down at the individual level, but it remains valid statistically.

II. BOHMIAN QUANTUM GEOMETRODYNAMICS

In this section we discuss the quantum geometrodynamics via the de-Broglie–Bohm interpretation. The Wheeler-De Witt (WDW) approach [3] for quantization of gravity is based on the 3+1 decomposition of space-time (ADM decomposition). This splitting is necessary because we use the canonical quantization of the Hamiltonian, and the Hamiltonian formulation is not covariant. In the ADM decomposition the space-time is decomposed into spatial slices which are labled by a time variable t (Σ_t). In this manner, the metric is specified by lapse (N) and shift (N_i) functions and the induced metric (h_{ij}). The line-element is:

$$ds^2 = (Ndt)^2 - h_{ij}(N^i dt + dx^i)(N^j dt + dx^j) \tag{1}$$

The extrinsic curvature of the spatial surfaces Σ_t's are given by:

$$K_{ij} = \frac{1}{2N}\left[\mathcal{D}_j N_i + \mathcal{D}_i N_j - \dot{h}_{ij}\right] \tag{2}$$

where the covariant derivative $\mathcal{D}_i$ is defined with respect to h_{ij} metric and the dot above letters represents time derivative. The Einstein-Hilbert action can be written as:

$$\mathcal{A} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\mathcal{R}$$

$$= -\frac{1}{16\pi G}\int d^3x\sqrt{h}N\left[K^2 - K_{ij}K^{ij} + \ {}^{(3)}\mathcal{R}\right] + \text{surface terms.} \tag{3}$$

where $K = trace(K_{ij})$, $h = det(h_{ij})$ and ${}^{(3)}\mathcal{R}$ is the three-dimmensional curvature. If one defines the conjugate momenta as usuall, we have:

$$\pi = \frac{\delta\mathcal{L}}{\delta\dot{N}} = 0 \tag{4}$$

$$\pi^i = \frac{\delta\mathcal{L}}{\delta\dot{N}_i} = 0 \tag{5}$$

$$\pi^{ij} = \frac{\delta \mathcal{L}}{\delta \dot{h}_{ij}} = \frac{\sqrt{h}}{16\pi G}\left(K^{ij} - K h^{ij}\right) \tag{6}$$

It is concluded that the N and N_i functions aren't dynamical and h_{ij} is the only dynamical degree of freedom. The equations (4) and (5) are primary constraints. The Hamiltonian is:

$$H = \int d^3x (N\mathcal{H}_G + N^i \mathcal{H}_i) \tag{7}$$

where

$$\mathcal{H}_G = 16\pi G G_{ijkl}\pi^{ij}\pi^{kl} - \frac{\sqrt{h}\ {}^{(3)}\mathcal{R}}{16\pi G} \tag{8}$$

$$\mathcal{H}^i = -\frac{1}{8\pi G}\mathcal{D}_j \pi^{ij} \tag{9}$$

$$G_{ijkl} = \frac{1}{2\sqrt{h}}\left(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl}\right) \tag{10}$$

Since the first constraints must be satisfied at all times, we must have:

$$\dot{\pi} = -\{H, \pi\} = \frac{\delta H}{\delta N} = 0 \Longrightarrow \mathcal{H}_G = 0 \tag{11}$$

$$\dot{\pi}_i = -\{H, \pi_i\} = \frac{\delta H}{\delta N^i} = 0 \Longrightarrow \mathcal{H}_i = 0 \tag{12}$$

These equalities are the secondary constraints, which are satisfied at all times because of the specific form of the Hamiltonian. Thus we have no new constraints anymore. In the canonical quantization, the relations (11) and (12) limit the physical state domain. In other words, among all the states in the Hilbert space, some special ones are physical. We distinguish these states by applying the constraints (11) and (12) weakly, i.e.:

$$\hat{\mathcal{H}}_G \Psi[h_{ij}] = 0 \tag{13}$$

$$\hat{\mathcal{H}}_i \Psi[h_{ij}] = 0 \tag{14}$$

where Ψ is the physical wavefunction of the universe.

According to the canonical quantization process, the $\hat{\mathcal{H}}_i$ and $\hat{\mathcal{H}}_G$ operators can be obtained by substituding the canonical momenta π^{ij} with $-i\frac{\delta}{\delta h_{ij}}$ in $\mathcal{H}^i$ and $\mathcal{H}_G$ which are the three and one-dimmensional diffeomorphism generators on the space-like surfaces and time-like direction, respectively. Then the wave-function of the universe must be annihilated by these generators. The constraint $\hat{\mathcal{H}}_G\Psi = 0$ (the Hamiltonian constraint) is the WDW equation. Its extension to the case in which the matter field ϕ exist, is:

$$\left[16\pi G h^{-q}\frac{\delta}{\delta h_{ij}}h^q G_{ijkl}\frac{\delta}{\delta h_{kl}} + \frac{\sqrt{h}}{16\pi G}\,{}^{(3)}\mathcal{R} - \mathcal{T}^0_0(\phi, -i\partial/\partial\phi)\right]\Psi[h_{ij}, \phi] = 0 \qquad (15)$$

where $\mathcal{T}_{\mu\nu}$ is the matter field energy-momentum tensor, and q is the ordering parameter. This equation and:

$$\hat{\mathcal{H}}_i\Psi = 0 \Longrightarrow -\frac{1}{8\pi G}\mathcal{D}_j\frac{\delta\Psi}{\delta h_{ij}} + \partial^i\phi\frac{\delta\Psi}{\delta\phi} = 0 \qquad (16)$$

specify the wavefunction of the universe.

The WDW equation has the following points:

- *The time parameter which defines the foilation of the space-time, doesn't appear in it. (the so-called time-problem in quantum gravity)*
- *A different ordering of factors leads to a different result.*
- *In practice, for solving the WDW equation, instead of using an infinite-dimmensional superspace, we must limit ourselves to a mini-superspace in which some of the degrees of freedom are non-frozen.*
- *It is necessary for the wave-function to be square-integrable, in order to have a probabilistic interpretation for it. But this is not possible for all cases, because a precise definition of the inner product is not known in quantum gravity.*

- *The WDW equation contains a multiplication of two functional derivatives which are calculated at the same point. Then applying the WDW operator on Ψ, we have a multiplication of two delta functions at one point. Therefore we must regularize the kinetic term of the WDW equation as:*

$$h^{-q}\frac{\delta}{\delta h_{ij}(\vec{x})}h^q G_{ijkl}\frac{\delta\Psi}{\delta h_{kl}(\vec{x})} \longrightarrow \underbrace{h^{-q}\frac{\delta}{\delta h_{ij}(\vec{x})}h^q\tilde{G}_{ijkl}(\vec{x},\vec{x}';t)\frac{\delta\Psi}{\delta h_{kl}(\vec{x}')}}_{\Delta_{reg.}} \tag{17}$$

  where $\lim_{t\to 0}\tilde{G}_{ijkl}(\vec{x},\vec{x}',t) = G_{ijkl}\delta(\vec{x}-\vec{x}')$ *and* $\tilde{G}_{ijkl}$ *satisfies the heat equation (heat kernel). The physical wave-functions must be annihilated by constraints. Thus:*

$$\Psi_{physical} = \delta(\mathcal{H}_G^{reg.})\Psi[h_{ij}] = \int DM(x)e^{i\int d^3xM(x)\mathcal{H}_G^{reg.}}\Psi[h_{ij}] \tag{18}$$

- *In the classical limit, we have:*

$$\{\mathcal{H}_i, H\} = \{\mathcal{H}_G, H\} = 0 \tag{19}$$

  where $\{,\}$ *represents the poisson bracket. This means that* $\mathcal{H}_i$ *and* $\mathcal{H}_G$ *form a closed algebra, and no new constriant appears. But this fact is problematic at the quantum level.*

Now, we use the canonical transformation $\Psi(h_{ij}) = \Gamma(h_{ij})e^{iS(h_{ij})}$ *in the equations (15) and (16), ignoring the matter fields for simplicity. Equating the real and imaginary parts of the equation (15), one gets:*

$$16\pi G\tilde{G}_{ijkl}\frac{\delta S}{\delta h_{ij}}\frac{\delta S}{\delta h_{kl}} - \frac{\sqrt{h}}{16\pi G}({}^{(3)}\mathcal{R} - Q_G) = 0 \tag{20}$$

$$\frac{\delta}{\delta h_{ij}}\left[h^q\tilde{G}_{ijkl}\frac{\delta S}{\delta h_{kl}}\Gamma^2\right] = 0 \tag{21}$$

where Q_G *is the quantum potential of the gravitational field:*

$$Q_G(h_{ij}) = -\frac{1}{\sqrt{h}\Gamma}\left(\tilde{G}_{ijkl}\frac{\delta^2\Gamma}{\delta h_{ij}\delta h_{kl}} + h^{-q}\frac{\delta h^q \tilde{G}_{ijkl}}{\delta h_{ij}}\frac{\delta\Gamma}{\delta h_{kl}}\right) \tag{22}$$

Also equation (16) reads as:

$$\mathcal{D}_j\frac{\delta\Gamma}{\delta h_{ij}} = 0 \tag{23}$$

$$\mathcal{D}_j\frac{\delta S}{\delta h_{ij}} = 0 \tag{24}$$

Equation (20) is a modified Hamilton-Jacobi equation. It indicates that the only difference between classical and quantum universes is the existance of the quantum potential in the latter. Equation (21) shows the conservation of the probability in the superspace. In addition, in the de-Broglie–Bohm theory, the guiding formula defines the momenta corresponding to the coordinates. Then, for the dynamical coordinates h_{ij} we have:

$$\pi^{kl} = \frac{\delta S}{\delta h_{kl}} = \frac{\sqrt{h}}{16\pi G}(K^{kl} - h^{kl}K) \tag{25}$$

Here it must be noted that the Hamilton-Jacobi equation indicates the dynamical properties. So that one can deal with the Hamilton equations of motion instead of equations (20)-(25). So we have, equivalently, the following relations:

$$\dot{\pi}_{ij} = \{\pi_{ij}, \tilde{H}\} \tag{26}$$

$$\dot{h}_{ij} = \{h_{ij}, \tilde{H}\} \tag{27}$$

where

$$\tilde{H} = \int d^3x(N\tilde{\mathcal{H}}_G + N^i\tilde{\mathcal{H}}_i) \tag{28}$$

$$\tilde{\mathcal{H}}^i = -\frac{1}{8\pi G}\mathcal{D}_j\pi^{ij} \tag{29}$$

$$\tilde{\mathcal{H}}_G = 16\pi G\tilde{G}_{ijkl}\pi^{ij}\pi^{kl} - \frac{\sqrt{h}}{16\pi G}({}^{(3)}\mathcal{R} - Q_G) \tag{30}$$

The de-Broglie–Bohm approach has the following advantages:

- *Although the time parameter does not appear in the wave-function, it emerges from the guiding formula, naturally.*

- *In this theory the role of the wave-function is different from the Copenhagen quantum mechanics. Its phase indicates the evolution of the dynamical variables according to the guiding formula. Its amplitude charactrizes the quantum potential which includes all the quantum effects. These two specifications of the wave-function have appeared for the individual processes and the wave-function may be non-normalized. The other aspect of the wave-function appears at the statistical level. The square of its amplitude has the probability interpretation as in the Copenhagen quantum mechanics, and therefore it is necessary to be normalized.*

- *An important problem, with which we are concerned, is the role of the* $\mathcal{H}_i$ *and* $\mathcal{H}_G$ *constraints at the quantum level. We shall deal with this point in the following section.*

III. QUANTUM POTENTIAL AND THE GENERAL COVARIANCE

In this section, we first review some viewpoints about the role of the constraints in quantum gravity:

- *Gilkman [4] has used the de-Broglie–Bohm approach. Because of existance of the quantum potential term in* $\mathcal{H}_G$*, he has shown that the constraints' algebra is not closed. Therefore in order for the constraints to remain valid at all times, one obtains some new constraints, etc. Consequently, he believes that the symmetry given by the four dimmensional diffeomorfism dosen't exist for individual processes, after quantization.*

In his view, this point is related either to the existance of a minimal length in quantum gravity, or probably to the use of the ADM decomposition for quantization.

- *Shtanov [5] has pointed out the problem of constraints. He believes that in the classical mechanics the choice of a Lagrange multiplier does not have any effect on the physical solution. But in quantum gravity, using the de-Broglie–Bohm approach, the situation is different. From the guiding formula for $g_{\mu\nu}$ one sees that the role of N_i in the quantum dynamics is the same as in classical dynamics. But the N function plays different roles in the classical and quntum domains. After quantization, a non-local function (quantum potential) appears in H_G and causes the physical charactristics of $g_{\mu\nu}$ to depend on N, in the general case. In the classical limit, one ignores the quantum potetial in comparison with the classical potentials. In this limit, the dependence is removed.*

  Shtanov concludes that the quantum dynamics of gravity breaks down the foliation-invariance of the classical general relativity. This is a result of the quantum non-locality. Of course, it must be noted that the foliation-invariance breaking only exists for the individual processes, according to the de-Broglie–Bohm theory. Because in this theory, it is not necessary for the dynamics of an individual system to follow all the statistical invariances.

- *Horiguchi et. al. [6] have first regularized the WDW eqation and then normalized it, by preserving the three-dimmensional general covariance. Therefore, the momenta-constraints algebra doesn't leads to some new constraints (anomali freedom of momenta constraints). Then, they have considered the Hamiltonian constraint algebra*

and indicated that an anomalous term may appear from the commutation between $\Delta_{reg.}$ and $\sqrt{h}\ {}^{(3)}\mathcal{R}$. If one sets this commutation relation equal to zero, a new constraint would result besides the WDW equation.

- *Blaut et. al. [7] have obtained the regularized WDW equation with an anomaly free condition for constraint algebra. They have observed that this condition is satisfied only for a specific subset of the wave-functions. This subset contains the wave-functions that are functions of 3-scalar densities. Then, they have used the quantum potential approach and shown that quantum gravity has less symmetry than classical gravity. The definition of N in the Hamiltonian is not free and it is fixed by the Hamiltonian constraint. In other words, the general covariance breaks down because of the non-existance of time translational symmetry at the individual level. This fact must have some physical effects at the Planck scale. In the classical limit, where we ignore the quantum effects, the time translational symmetry would be restored.*

From the foregoing discussion, we can deduce an important result: *It is possible to find some wave-function for which the constraints' algebra is satisfied, using the regularized WDW equation.* Thus, from the Bohmian point of view, the constraints can be met statistically, but not necessarily individually.

In this paper, our aim is to discuss explicitly the break down of the constraints in confirmation of the above result. It is a well-known fact that four equations of the Einstein's equations are constraints on the extrinsic curvature (K^{ij}) and the 3-space metric, and the remining equations represent the time evolution of the 3-space metric. This point results from the fact that some of the Reimann tensor components depends only on the extrinsic

curvature and the 3-space intrinsic curvature.

From the Gauss-Codadzi equations [8] we have:

$$\mathcal{R}^0_{ijk} = \mathcal{D}_i K_{jk} - \mathcal{D}_j K_{ik} \tag{31}$$

$$\mathcal{R}^m_{ijk} = \ {}^{(3)}\mathcal{R}^m_{ijk} + K_{jk}K^m_i - K_{ik}K^m_j \tag{32}$$

Using these, one obtains:

$$G^0_0 = -\frac{1}{2}\left[\ {}^{(3)}\mathcal{R} + K^2 - K_{ij}K^{ij}\right] \tag{33}$$

Thus, one of the Einstein's equations relates the extrinsic curvature of the space-like slices and their intrinsic scalar curvature. Furthurmore, one other result from the equations (31) and (32) is:

$$G^0_i = \mathcal{D}_j K^j_i - \mathcal{D}_i K \tag{34}$$

Therefore, these three Einstein's equations are constraints on the extinsic curvature of the space-like surfaces.

After quantization, the constraints (33) and (34) would appear in the form of equations (15)-(16) in the absence of the matter field or (20)-(24), from the Copenhagen or Bohmian points of view, respectively. Starting from the Bohmian form, the relation (20) gives:

$$16\pi G \tilde{G}_{ijkl}\pi^{ij}\pi^{kl} - \frac{\sqrt{h}}{16\pi G}(\ {}^{(3)}\mathcal{R} - Q_G) = 0 \tag{35}$$

On using the equation (25) and the fact that $G_{ijkl}\pi^{kl} = K_{ij}/16\pi G$, we have:

$$(K^{ij} - Kh^{ij})K_{ij} - \ {}^{(3)}\mathcal{R} + Q_G = 0 \tag{36}$$

Thus:

$$G^0_0 = -\frac{Q_G}{2} \tag{37}$$

Therefore taking the quantum effects into consideration, the constraint $G^0_0 = 0$ *would be corrected according to the relation (37). The other three constraints at the quantum level can be obtained from equation (24):*

$$\mathcal{D}_j \pi^{ij} = 0 \tag{38}$$

Since $\mathcal{D}_j h^{ij} = 0$*, we have:*

$$\mathcal{D}_j K^j_i - \mathcal{D}_i K = 0 \tag{39}$$

Therefore:

$$G^0_i = 0 \tag{40}$$

Thus the three-dimmensional diffeomorphism constraints are not changed when one takes the quantum effects into account. This fact can be also concluded by observing the absence of the quantum potential in the relation (24).

As we argued previously, the role of the momenta constraints for the classical and quantum cases is the same. But the Hamiltonian constraint for the quantum case is corrected by the quantum potential.

Now, we must obtain the corrections of the dynamical Einstein's equations (i.e the spatial components of $G_{\mu\nu}$*). These are the governing equations on the metric of the space-like surfaces* h_{ij} *(i.e. the equations of motion of* h_{ij}*). By noting to the relations (3) and (33), the Einstein-Hilbert action, in the ADM decomposition can be written as:*

$$\mathcal{A} = \frac{1}{8\pi G} \int d^4x N \sqrt{h} G^0_0 + \text{surface terms} \tag{41}$$

By varying $\mathcal{A}$ with respect to h_{mn}, one obtains:

$$G^{mn}(x) = -2\int d^4x' N \frac{\delta}{\delta h_{mn}(x)}(\sqrt{h(x')}G_0^0(x')) \tag{42}$$

So that if the Hamilton-Jacobi equation be multiplied by N and varied with respect to h_{mn}, the equation of motion of the 3-space metric would be obtained. Thus, substitution from (37) yields:

$$G^{mn}(x) = \int d^4x' N \frac{\delta}{\delta h_{mn}(x)}[\sqrt{h(x')}Q_G(x')] \tag{43}$$

This can also be deduced from the Hamilton-Jacobi equation. We know from the Hamilton-Jacobi formalism that by varying the Hamilton-Jacobi equation with respect to any coordinate and using the guiding formula, the equation of motion of that coordinate would be obtained. Therefore, varying the relation (35) and doing some algebra, leads to the relation (43).

Thus, the dynamical equations of the space-like surface metric are also corrected by the quantum potential.

IV. OBSERVATIONS

Now we are ready to discuss some important results:

- *In the Bohmian quantum theory of gravity, the general covariance, represented by $G_0^0 = 0$ and $G_i^0 = 0$ constraints, breaks down. This is because of the breaking of $G_0^0 = 0$ constraint for individual processes. The equations (37), (40) and (42) are not covariant and involve the spatial and time-like components differently. The Break down of the general covariance principle is caused by the quantum potential and shows*

that the equivalence principle is not valid for individual processes at quantum level, necessarily. Since in one sense, the equivalence principle is in contradiction to Mach's principle (i.e. in a local inertial frame, the laws of motion are independent of the distant matter), the break down of the former may be a step towards the latter. This point is discussed in the ref. [9] in detail.

- *According to the equations (37), (40) and (42), the modified Einstein's equations, are functianals of h_{ij} and the scalar Γ. This is suggesting that it is probably necessary to use a scalar-tensor theory for the quantum description of gravity.*

- *Although we started from a pure gravity field, in the general case with a matter field, one can do in a similar way and obtain the modified Einstein's equations as:*

$$G_0^0 = \kappa \mathcal{T}_0^0 - \frac{Q_G + Q_M}{2} \tag{44}$$

$$G_i^0 = \kappa \mathcal{T}_i^0 \tag{45}$$

$$G^{mn} = \kappa \mathcal{T}^{mn} + \int d^4x' N \frac{\delta}{\delta h_{mn}} [\sqrt{h}(Q_G + Q_M)] \tag{46}$$

where Q_M is the quantum potential of the matter resulted from the dependence of Γ upon the matter field and is independent of the ordering parameter. For example, for a scalar field, we have:

$$Q_M = -\frac{1}{h}\frac{1}{\Gamma}\frac{\delta^2\Gamma}{\delta\phi^2} \tag{47}$$

- *In Bohm's theory, the Hamilton-Jacobi equation and the Newton's equation of motion are respectively:*

$$\frac{\partial S}{\partial t} + \frac{|\vec{\nabla} S|^2}{2m} + V + Q = 0 \tag{48}$$

$$\frac{d\vec{p}}{dt} = -\vec{\nabla}(V + Q) \tag{49}$$

where Q is the quantum potential. Therefore in the classical limit, it is necessary that Q be numerically negligible ($Q <<$ other energies in Hamilton-Jacobi equation) and slowly varying (at least its first derivatives should be negligible: $\vec{\nabla} Q << \vec{\nabla} V$). Now, in the modified Einstein's equations, relations (44) and (46) are the Hamilton-Jacobi equation and the modified equation of motion. Thus in the classical limit, we ignore the second terms in the right hand side of these equations in comparison to the first term, and then the conventional Einstein's equations are obtained explicitly.

- One way to preserve the general covariance principle for the individual processes is to see the quantum potential as an agent for the quantum force. Just as in the nonrelativistic Bohm's theory we correct the Newton's second law as the relation (49), one can correct the Einstein's equations similarly. In a covariant way, one may write:

$$G^{\mu\nu} = \kappa \mathcal{T}^{\mu\nu} + F_Q^{\mu\nu} \tag{50}$$

  where $F_Q^{\mu\nu}$ is a second rank (quantum force) tensor, appeared at the right hand side as a quantum source for gravity.

  It must be pointed out that this equation is a new extension of the de-Broglie–Bohm theory for gravity. Naturally, its results are not necessarily the same as the Copenhagen quantum gravity or even the conventional de-broglie–Bohm theory of gravity. But this extension must be such that in some special limit, the usuall quantum gravity would result.

- From every aspect, the quantum potential is very important in quantum gravity. In reference [10], it is shown that the existance of the matter quantum potential is equivalent to introduction of a conformal factor in the space-time metric. This means that

the quantal effects of the matter may be thought as some geometrical effects. This viewpoint about the quantum potential has many advantages. For example it is able to remove the cosmological singularities in the early universe. [10]

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